

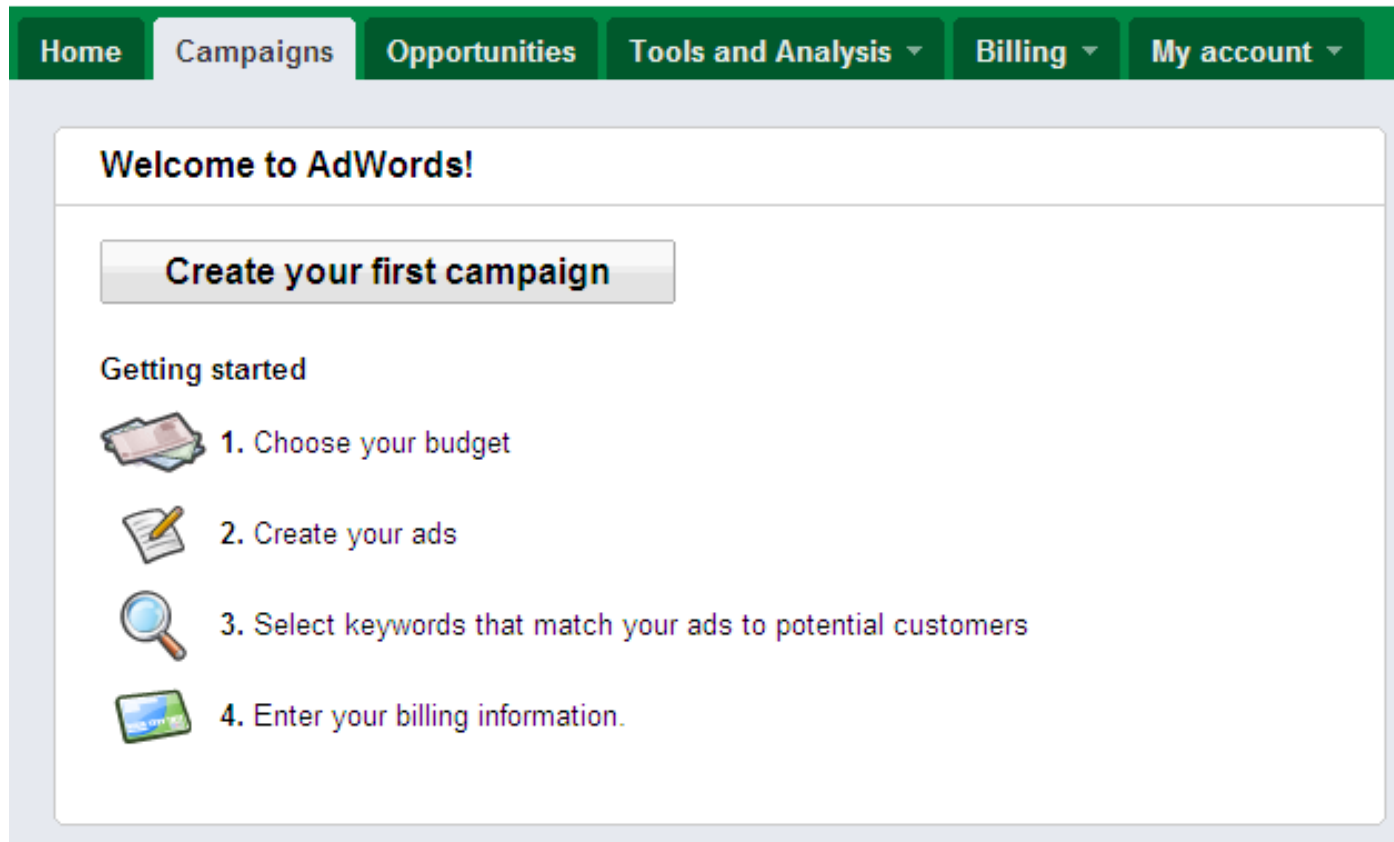
Polyhedral Clinching Auctions and the AdWords Polytope

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Joint work with Gagan Goel and
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Creating an Ads campaign ...

Google AdWords



The screenshot shows the Google AdWords interface. At the top, there is a green navigation bar with the following tabs: Home, Campaigns (selected), Opportunities, Tools and Analysis (with a dropdown arrow), Billing (with a dropdown arrow), and My account (with a dropdown arrow). Below the navigation bar, a white box contains the text "Welcome to AdWords!". Underneath this, there is a prominent button that says "Create your first campaign". Below the button, the section "Getting started" is followed by a numbered list of four steps, each with a small icon: 1. Choose your budget (with a stack of money icon), 2. Create your ads (with a notepad and pencil icon), 3. Select keywords that match your ads to potential customers (with a magnifying glass icon), and 4. Enter your billing information. (with a credit card icon).

Creating an Ads campaign ...





Google AdWords

Home Campaigns Opportunities Tools and Analysis Billing My account

Welcome to AdWords!

Create your first campaign

Getting started

-  1. Choose your budget
-  2. Create your ads
-  3. Select keywords that match your ads to potential customers
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Creating an Ads campaign ...



Bidding and budget

Bidding option [?](#) [Basic options](#) | [Advanced options](#)

I'll manually set my bids for clicks

AdWords will set my bids to help maximize clicks within my target budget

CPC bid limit [?](#) \$

Budget [?](#) \$ per day

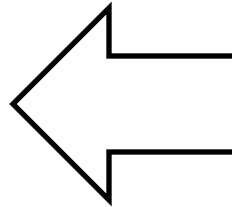
Actual daily spend may vary. [?](#)

How to deal with budgets in practice ?

VCG, GSP, ...

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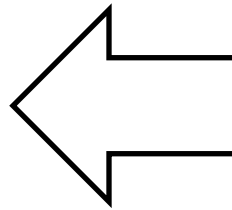
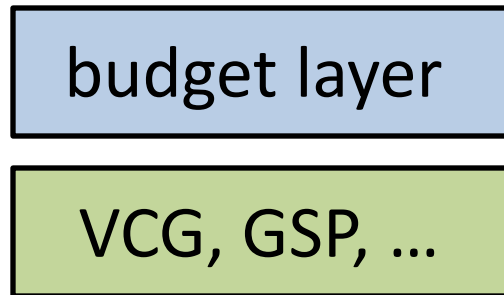
nice and well studied
auction with good
game-theoretic properties
but without budgets...

How to deal with budgets in practice ?

budget layer

VCG, GSP, ...

How to deal with budgets in practice ?



engineering fix to adapt the original auction to the budgeted setting. Original game theoretic analysis is now lost.

How to deal with budgets in practice ?

budget layer

VCG, GSP, ...

How to deal with budgets in practice ?

Polyhedral
Clinching
Auction

Goal:

Design an auction for AdWords that supports budgets natively, i.e., budgets are built in the game theoretic analysis

What do we mean by budgets ?

Classical quasi-linear utility function:

$$u_i = v_i(\text{vase}) - p_i$$

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$$u_i = v_i(\text{vase}) - p_i, \quad \text{if } p_i \leq B_i$$
$$= -\infty, \quad \text{o.w.}$$

Classical quasi-linear utility function:

$$u_i = v_i(\img alt="A blue and white floral vase" data-bbox="345 195 408 328")) - p_i$$

Very well understood: **VCG**, affine maximizers, ...

Budget constrained utility function:

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Surprisingly little is known.

Goal: Design auctions for budget
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(feasible set)

Desirable properties

- Incentive Compatibility:

$$v_i x_i(v_i, v_{-i}) - p_i(v_i, v_{-i}) \geq v_i x_i(v'_i, v_{-i}) - p_i(v'_i, v_{-i})$$

assumption: budgets B_i are public

- Individual rationality: $v_i x_i(v_i, v_{-i}) - p_i(v_i, v_{-i}) \geq 0$
- Pareto optimality:

An outcome (x, p) is **Pareto-optimal** if there is no (x', p') such that $u'_i \geq u_i$, $\sum p'_i \geq \sum p_i$ and at least one of them is strict.

Our main contribution

Solve this problem for a large class of feasible sets \mathbf{P} .

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Show this is impossible to be extended to general polytopes.

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Show this is impossible to be extended to general polytopes.

Conjecture: scaled polymatroids are the largest class for which this is possible.
(we supply evidence for that)

What do we know about budgets?

[Dobzinski, Lavi, Nisan, FOCS'08]

:: auction for one divisible good

[Fiat, Leonardi, Saia, Sankowski, EC'11]

:: auction for matching markets

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:: auction for matching markets

based on the **clinching auctions framework**

[Ausubel, AER'97]

How does it fit in our goal ?

[Dobzinski, Lavi, Nisan, FOCS'08]

$\mathbf{P} = \{x \in \mathbb{R}_+^n; \sum_i x_i \leq 1\}$ **Uniform Matroid**

[Fiat, Leonardi, Saia, Sankowski, EC'11]

$\mathbf{P} =$ **Transversal Matroid**

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For AdWords and other more complicated markets, we need to solve it for more generic feasibility constraints \mathbf{P}

Our Results

We provide an auction with all the desirable properties for any polymatroid \mathbf{P} .

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- Incentive compatibility
- Individual Rationality
- Budget Feasibility
- Pareto Optimality

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We provide an auction with all the desirable properties for any polymatroid \mathbf{P} .

$$\mathbf{P} = \{x \in \mathbb{R}_+^n; \sum_{i \in S} x_i \leq f(S); \forall S \subseteq [n]\}$$

for a submodular function f .

Our Results

We provide an auction with all the desirable properties for any polymatroid \mathbf{P} .

Our auction only needs oracle access to the submodular function f .

Our auction has a natural geometric flavor.

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Many applications

Auctions for network design, queuing systems, video on demand, matching markets, internet advertisement, ...

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Our results

The set of (x_1, \dots, x_n) that can be obtained this way form a polymatroid. We call it the **AdWords Polytope**.

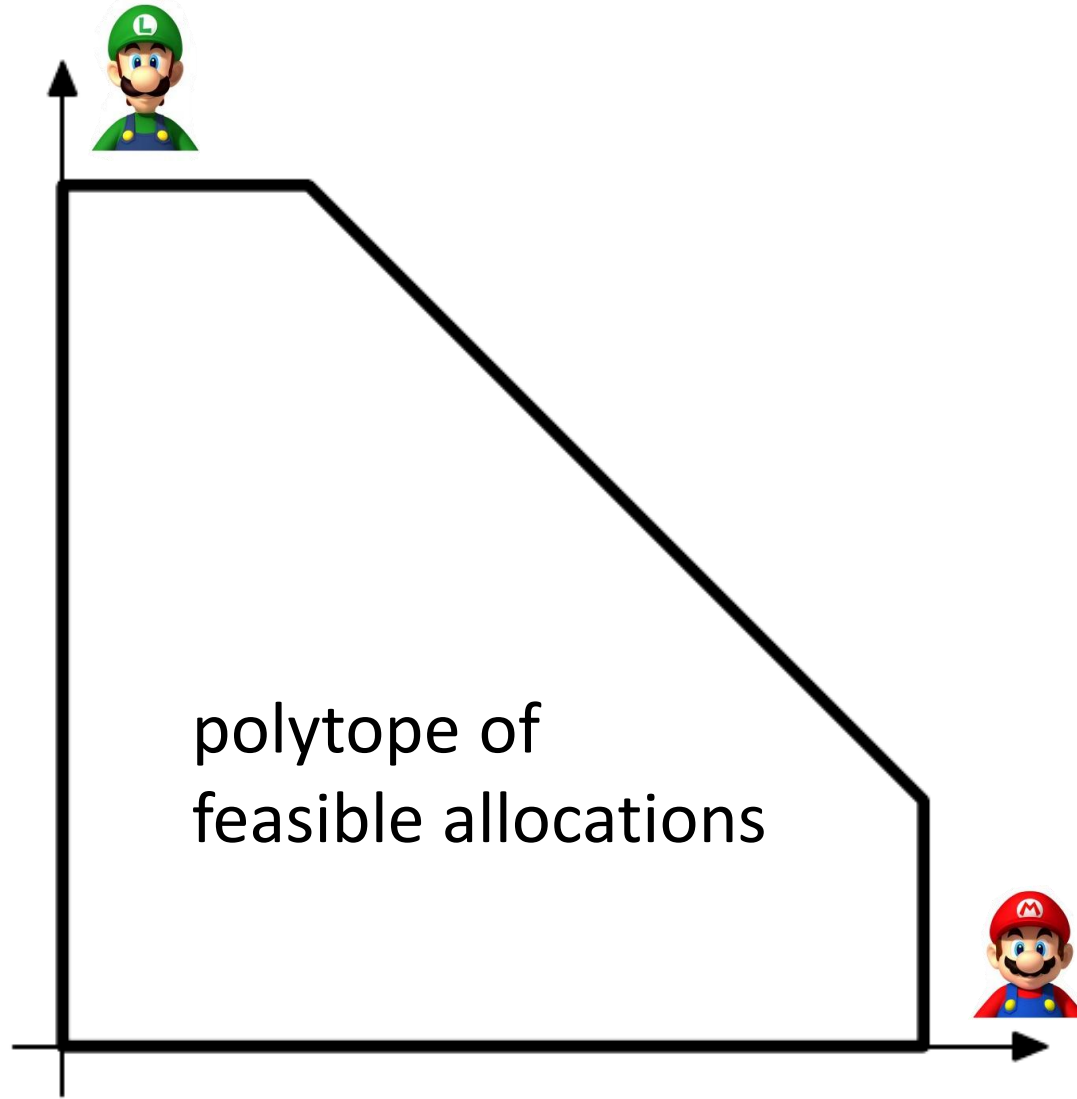
General model:

- multiple slots
- multiple keywords
- easy to generalize

Also on Sponsored Search with Budgets

Independently, [**Colini-Baldeschi, Henzinger, Leonardi, Starnberger, 2012**] design an auction for sponsored search with one keyword, multiple slots and budgets.

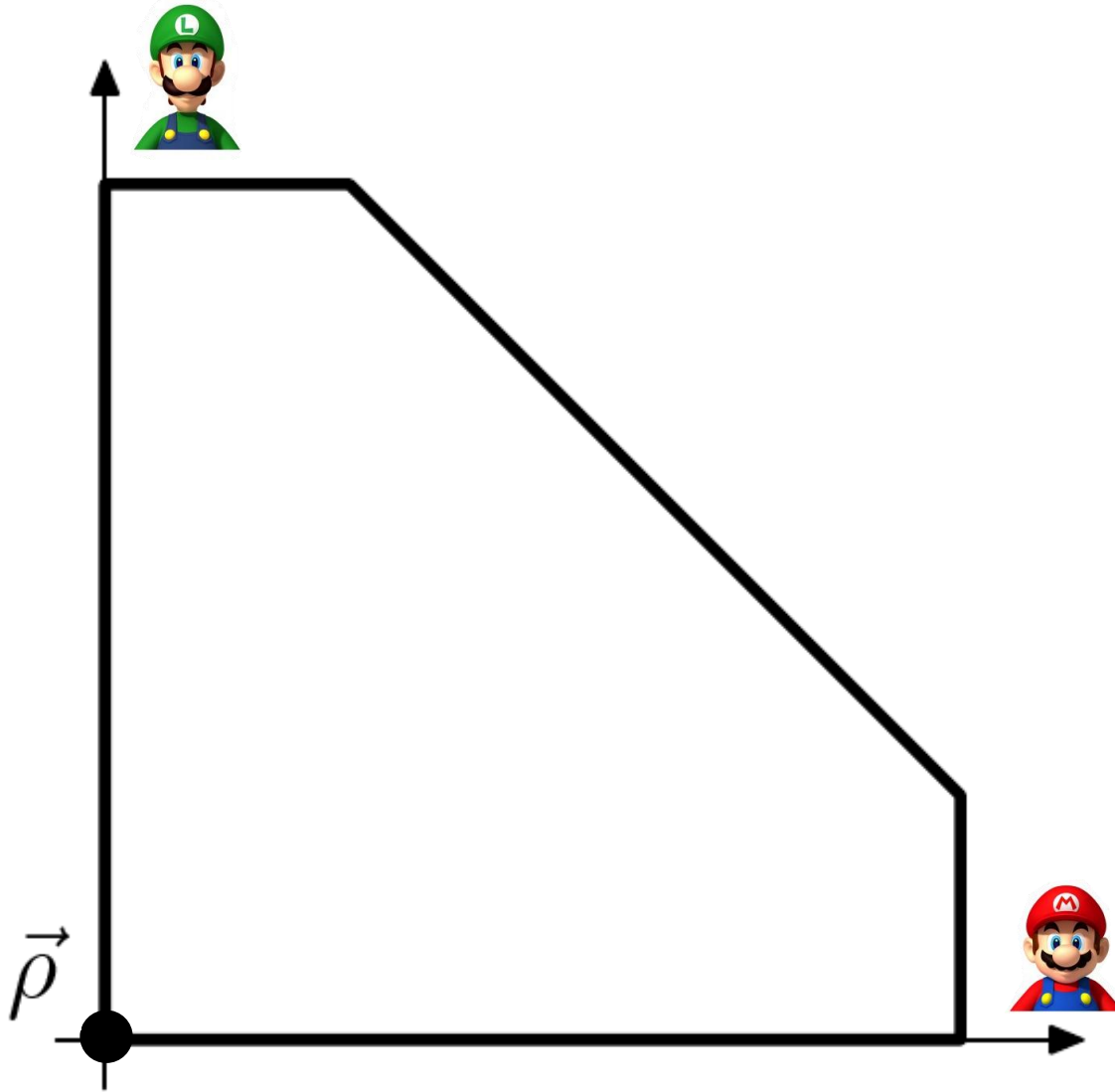
Our auction



price clock



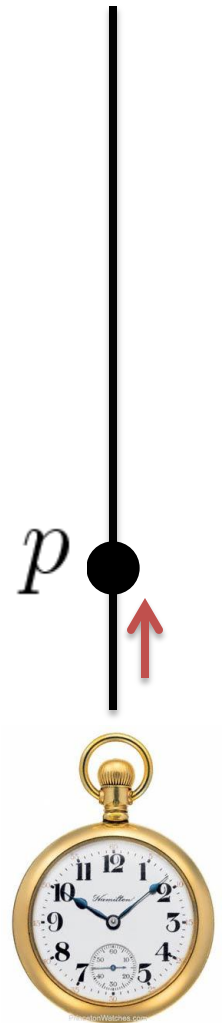
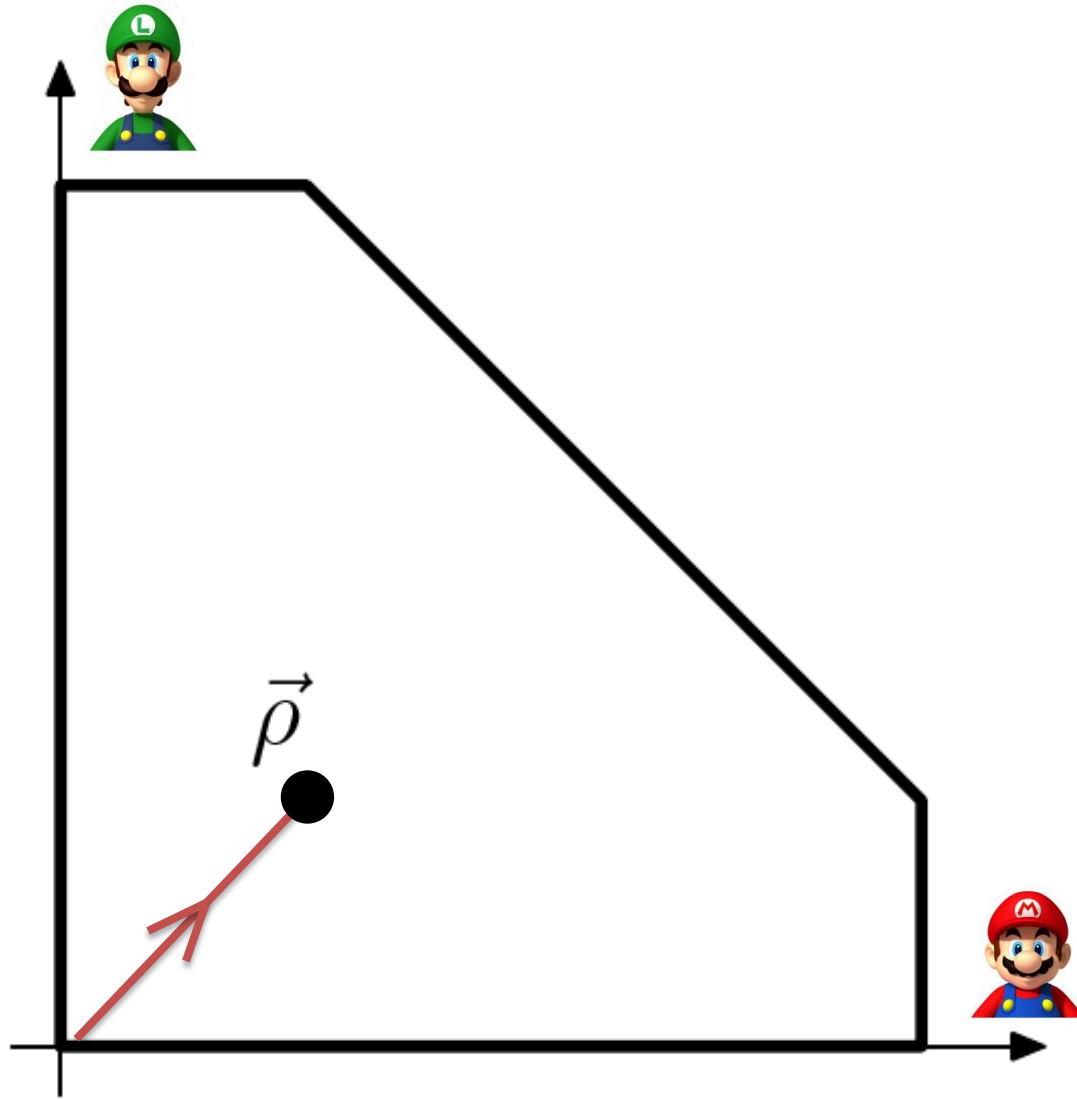
Our auction



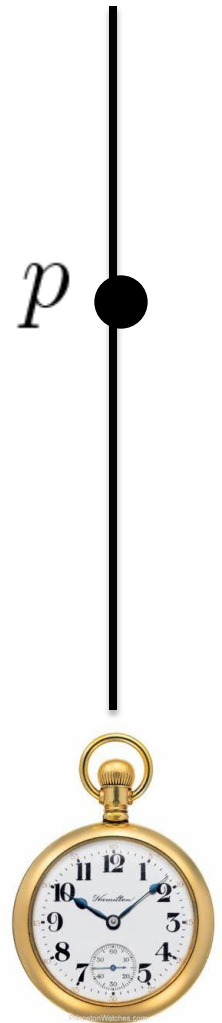
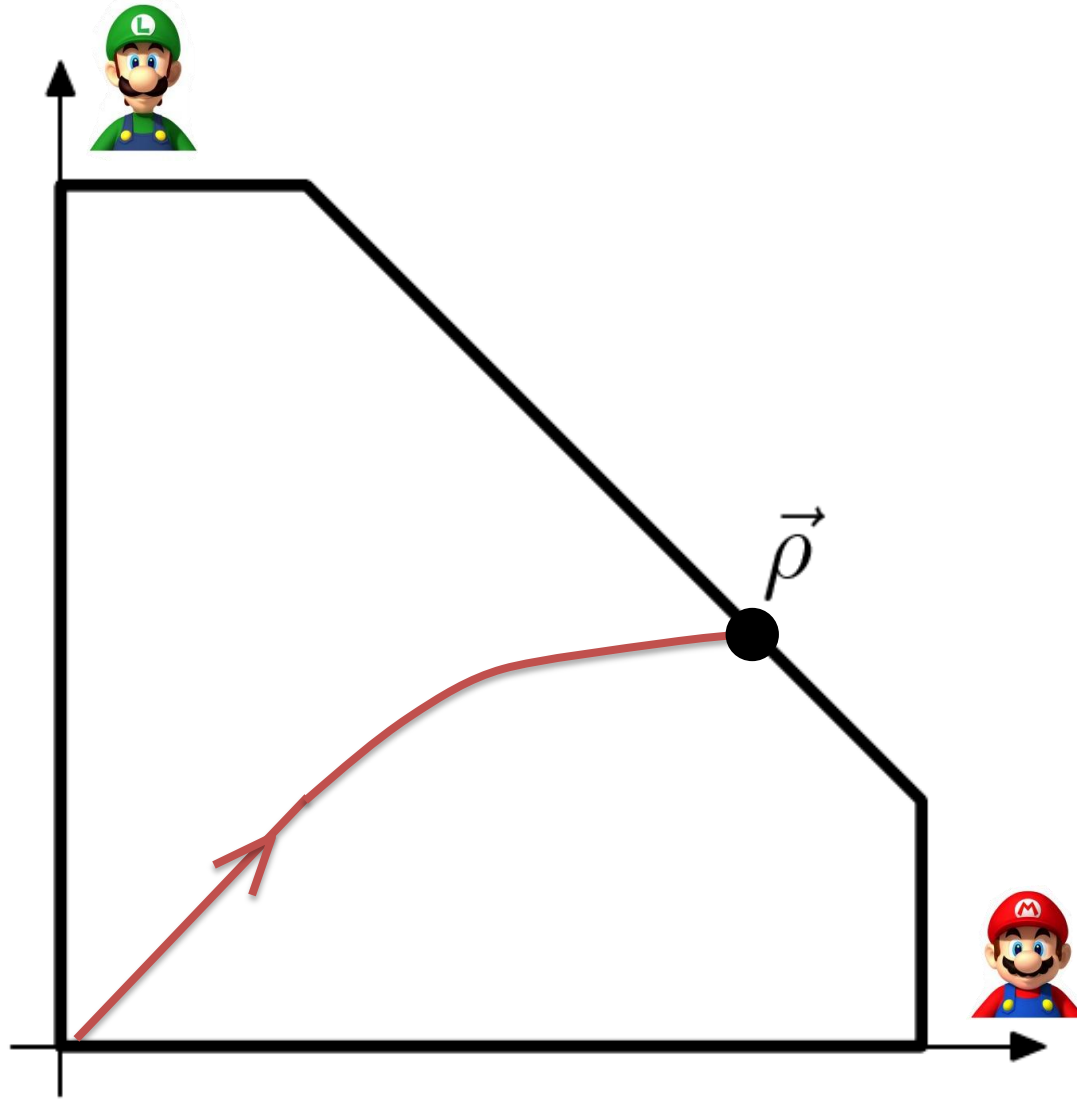
$$p = 0$$



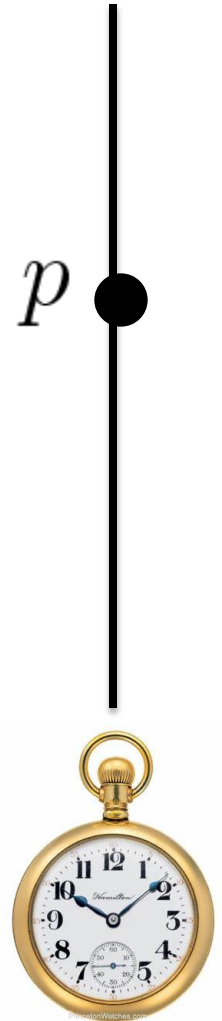
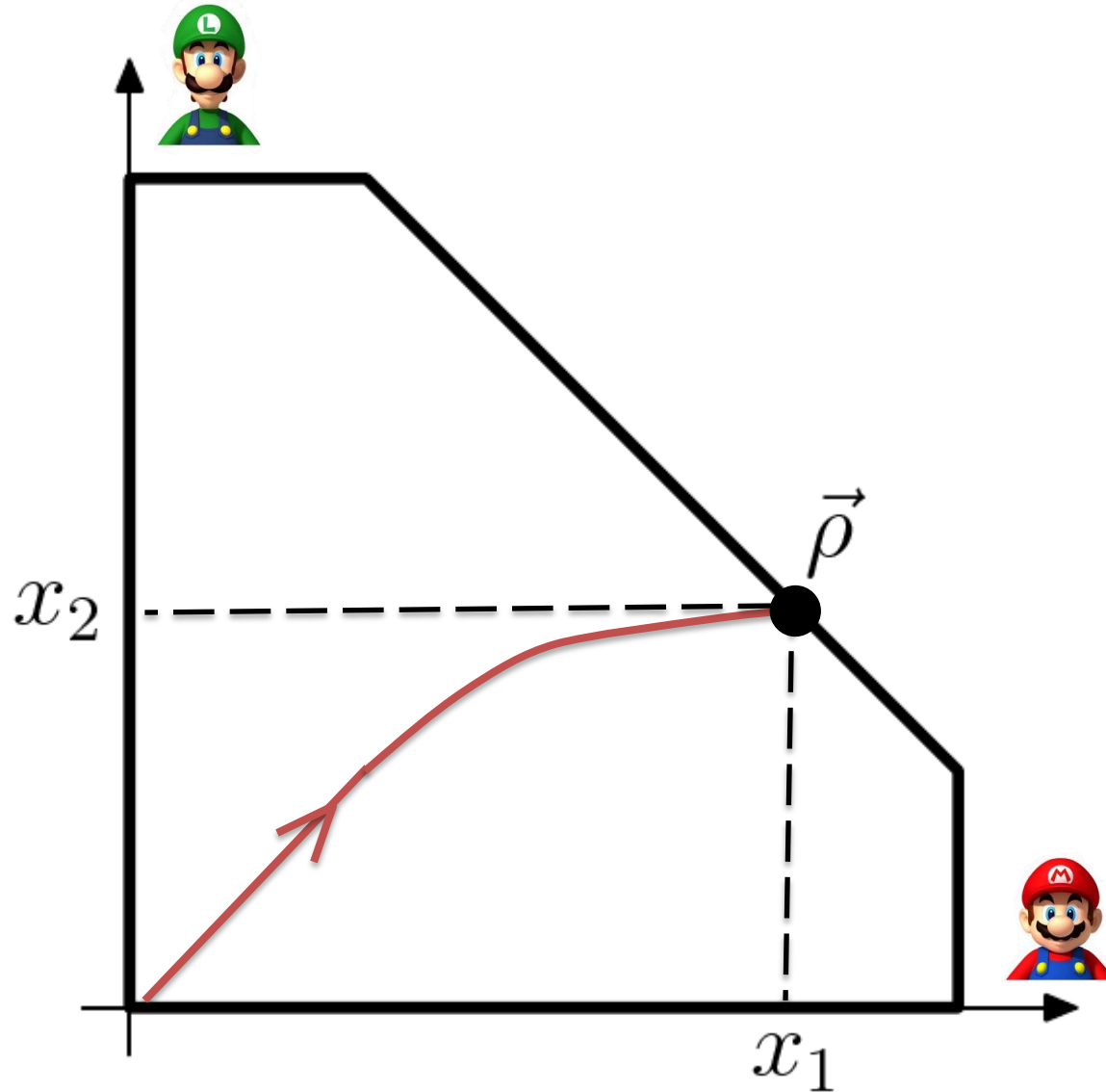
Our auction



Our auction



Our auction



Our auction

In each step compute demands d_i at price p

$$d_i = \frac{B_i^{\text{rem}}}{p} \text{ if } p \leq v_i ; \text{ and } 0 \text{ o.w.}$$

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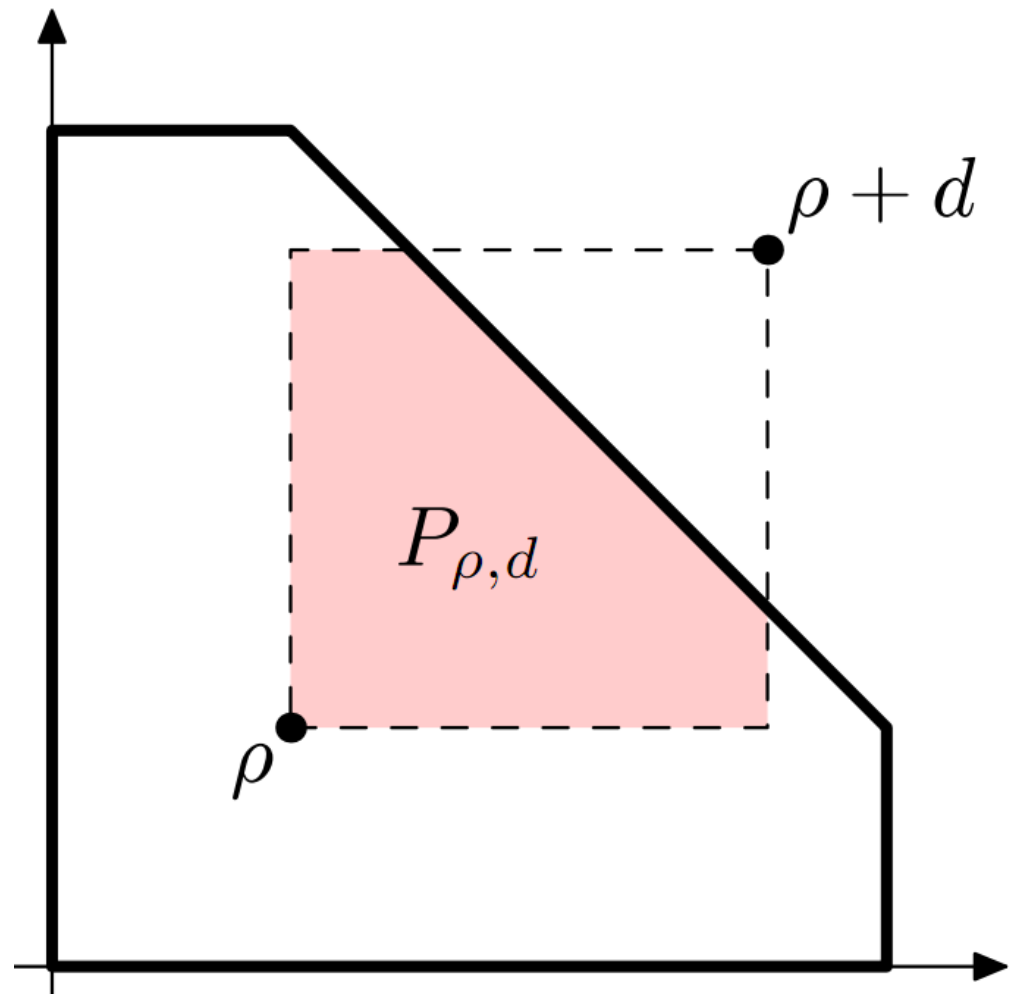
$$d_i = \frac{B_i^{\text{rem}}}{p} \text{ if } p \leq v_i ; \text{ and } 0 \text{ o.w.}$$

Compute **clinched** amount δ_i

$$\rho_i = \rho_i + \delta_i \quad B_i = B_i - p\delta_i$$

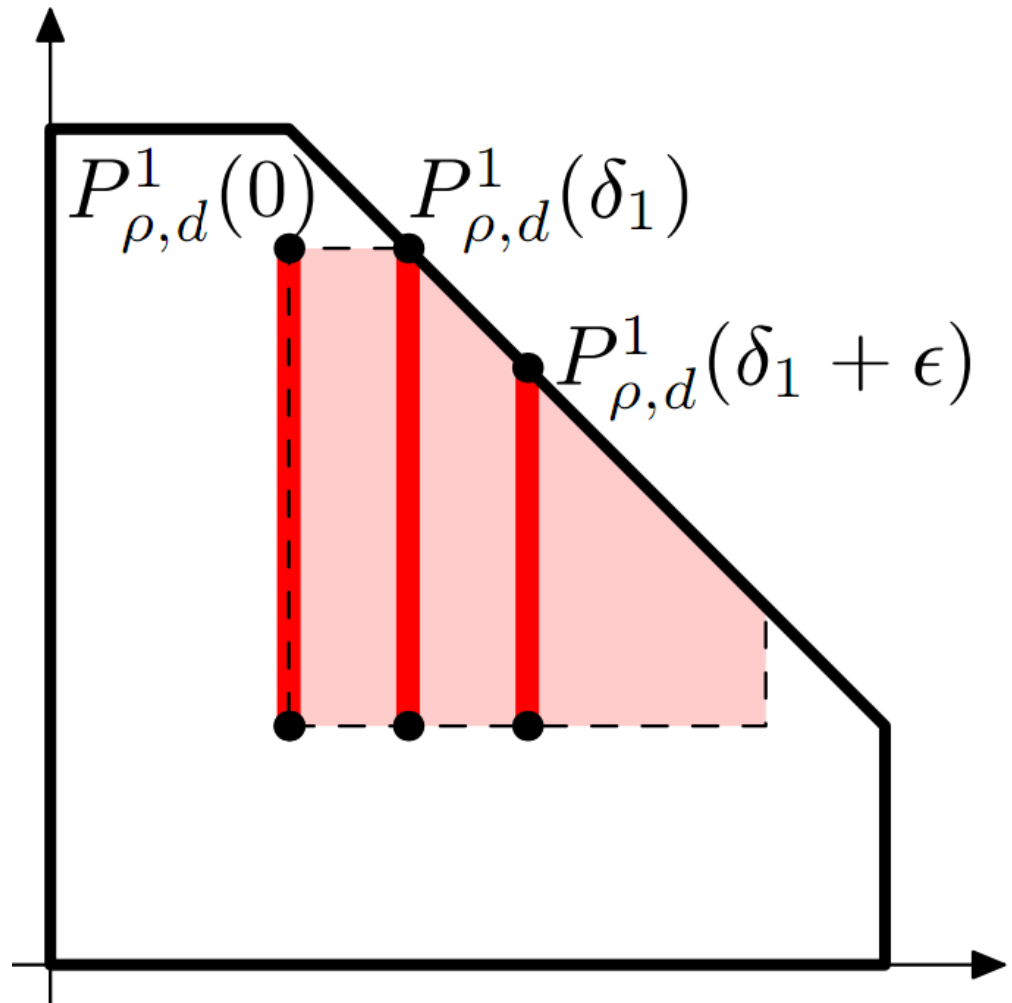
Computing clinched amounts δ_i

What is the allocations that are still feasible at this point?



Our auction: how to implement **clinch** ?

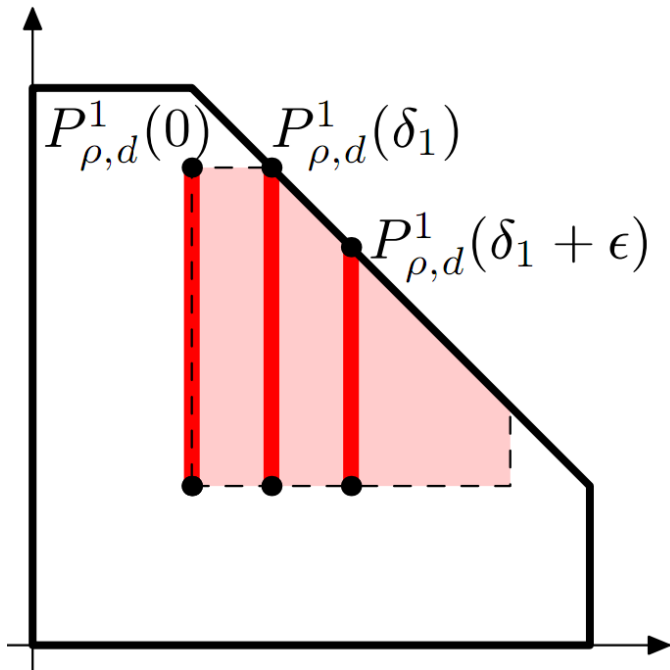
How much can I allocate to 1 without harming player 2?



Our auction: how to implement **clinch** ?

$$P_{\rho,d}^i(x_i) = \{x_{-i} \in \mathbb{R}_+^{[n] \setminus i}; (x_i, x_{-i}) \in P_{\rho,d}\}$$

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Theorem: Clinching as defined above results in a feasible allocation. If \mathbf{P} is a polymatroid, δ_i can be computed efficiently using submodular minimization.

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[in practice there are more efficient algorithms for each case]

Summary of the proof

- Show clinching is well-defined and can be computed efficiently
- Characterize Pareto-optimal outcomes for polymatroidal environments
- Show that the auction produces an outcome satisfying the characterization

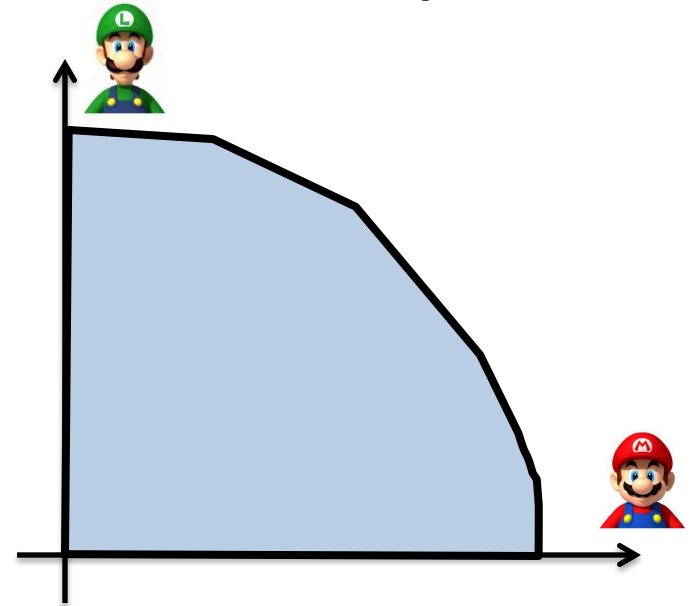
Extensions and Limits

Going beyond polymatroids...

General convex environment

One budget-constrained player

For a single budget constrained player (and many other unconstrained ones), it is possible to design an auction for any convex environment.



What about 2 budget constrained players ?

Weak impossibility: There is no auction following the clinching framework beyond (scaled) polymatroids.

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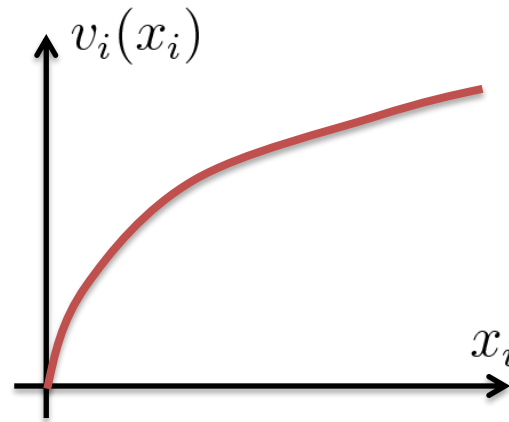
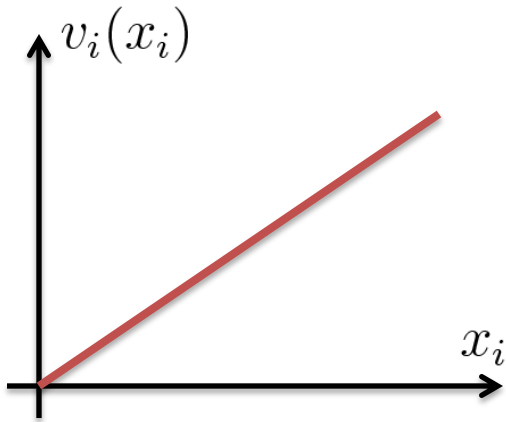
Stronger impossibility: There exists a class of polytopes, for which **no auction exists** satisfying all the desirable properties.

No hope of an auction for a general polyhedral environment.

Impossibility for decreasing marginals

Single divisible good: $\{x \in \mathbb{R}_+^n; \sum_i x_i \leq 1\}$

Decreasing marginal valuations



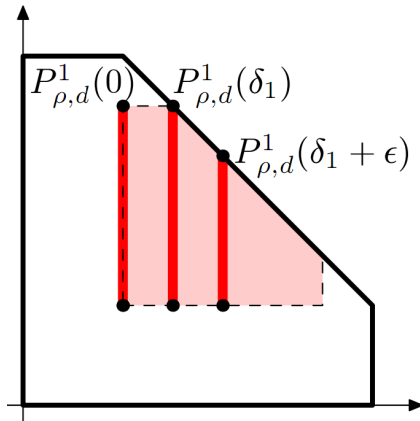
Impossibility for decreasing marginals

Single divisible good: $\{x \in \mathbb{R}_+^n; \sum_i x_i \leq 1\}$

Thm: No auction with all the desirable properties for one divisible good with decreasing marginals.

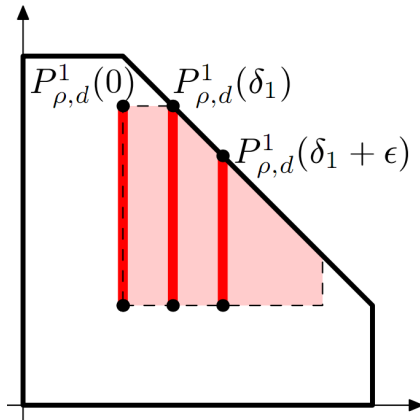
Strengthens previous impossibility results of [Lavi, May'11] and [Fiat et al'11]

Summary

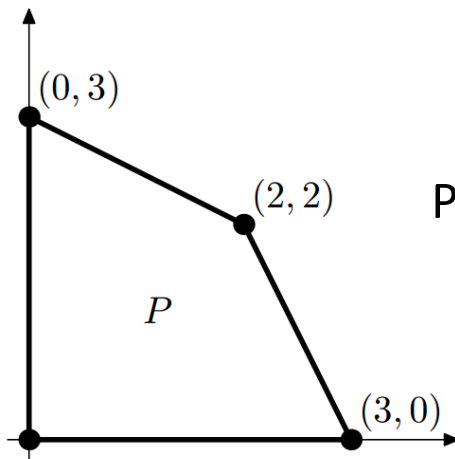


**Clinching auction for
polymatroids**

Summary

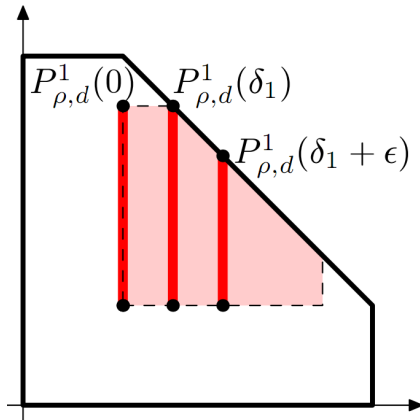


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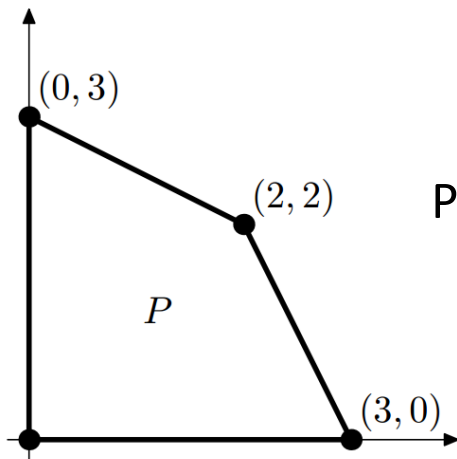


**Characterization of
Pareto Optimal Auctions
in general polyhedral
environments**

Summary



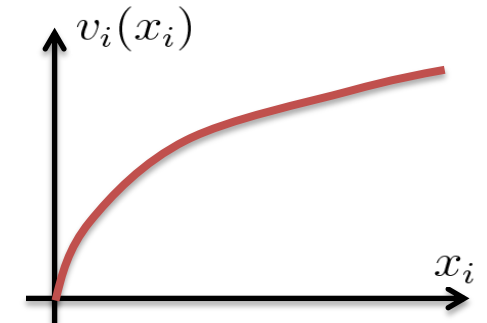
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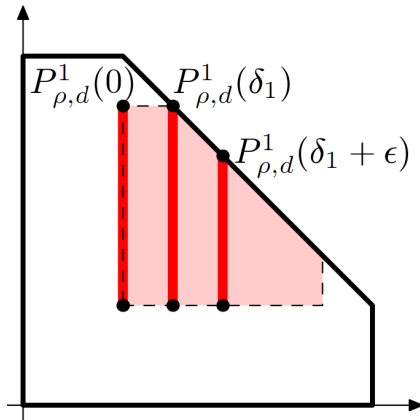


**Impossibility for
general polytopes**

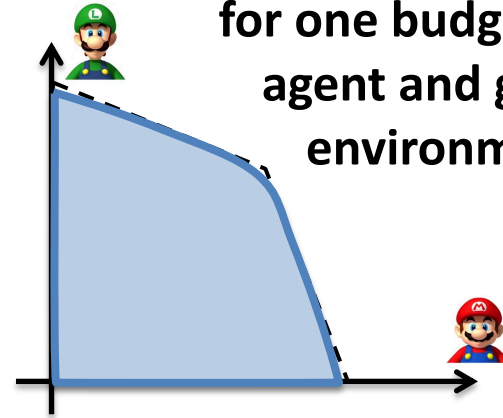


**Impossibility for
decreasing-marginals
and budgets**

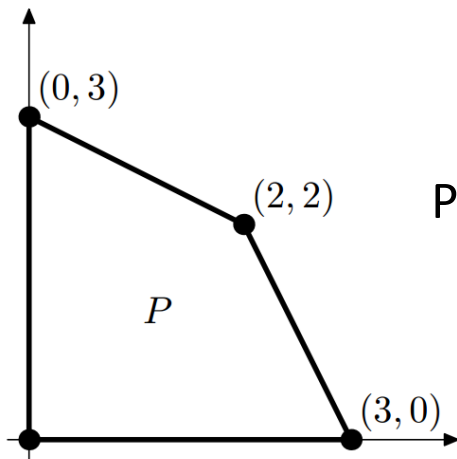
Summary



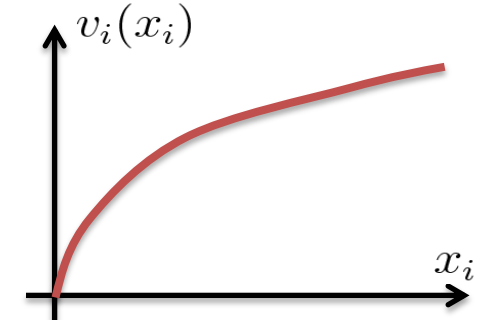
**Clinching auction for
polymatroids**



**Positive results
for one budget-constr
agent and general
environments**



**Characterization of
Pareto Optimal Auctions
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**Impossibility for
decreasing-marginals
and budgets**



Thanks !