

On Revenue in the Generalized Second Price Auction

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Two AdAuctions Mechanisms

Generalized Second
Price Auction

Vickrey-Clarke-Groves
Mechanism

Two AdAuctions Mechanisms

GSP

VCG

Google™

YAHOO!®

bing™



Two AdAuctions Mechanisms

GSP

VCG

Main Question:

How do those mechanisms compare from a **game-theoretic viewpoint** ?

Two AdAuctions Mechanisms

GSP

- sort by bid

VCG

- sort by bid

Two AdAuctions Mechanisms

GSP

- sort by bid
- pricing = next highest bid

VCG

- sort by bid
- complicated pricing rule

Two AdAuctions Mechanisms

GSP

- Simple
- Industrial standard

VCG

- Complicated
- Optimal in theory

With respect to which metric ?

With respect to which metric ?

Advertisers

Search
Engine

Users

With respect to which metric ?

Advertisers

Clicks * (value/click – pay/click)

Search
Engine

Clicks * pay/click

Users

Usefulness of ads

With respect to which metric ?

Advertisers

Clicks * (value/click – pay/click)

Search
Engine

Clicks * pay/click

Revenue

Users

Usefulness of ads

With respect to which metric ?

Advertisers

Clicks * (value/click – pay/click)

Search
Engine

Clicks * pay/click

Revenue
(our focus)

Users

Usefulness of ads

With respect to which metric ?

Advertisers

Clicks * (value/click – pay/click)

Search
Engine

Clicks * pay/click

Social Welfare = \sum Clicks . (value/click)

Users

Usefulness of ads

With respect to which metric ?

Advertisers

Clicks * (value/click – pay/click)

Search
Engine

Clicks * pay/click

Social Welfare = \sum Clicks . (value/click)



Users

Usefulness of ads

With respect to which metric ?

Advertisers

Clicks * (value/click – pay/click)

Search
Engine

Clicks * pay/click

Social Welfare = \sum Clicks . (value/click)

Users

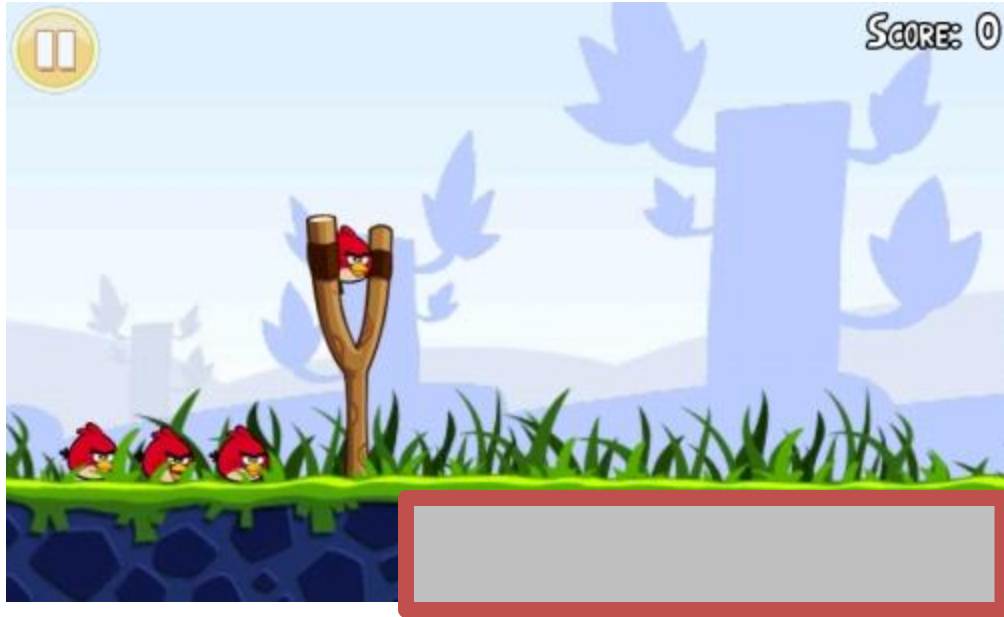
Usefulness of ads

[Varian], [Eldeman, Ostrovsky, Schwarz], [Paes Leme, Tardos], [Lucier, Paes Leme], [Caragiannis, Kaklamanis, Kanellopoulos, Kyropoulou], [CKKKLPLT], [Athey, Nekipelov], [Ellison, Athey], ...

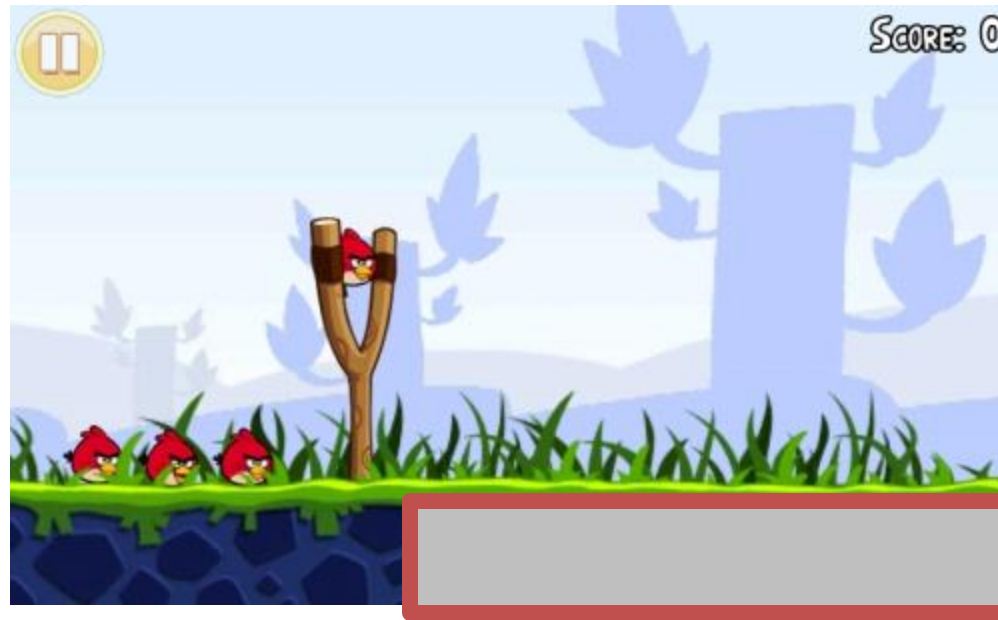
Warm-up: When we have one ad slot...



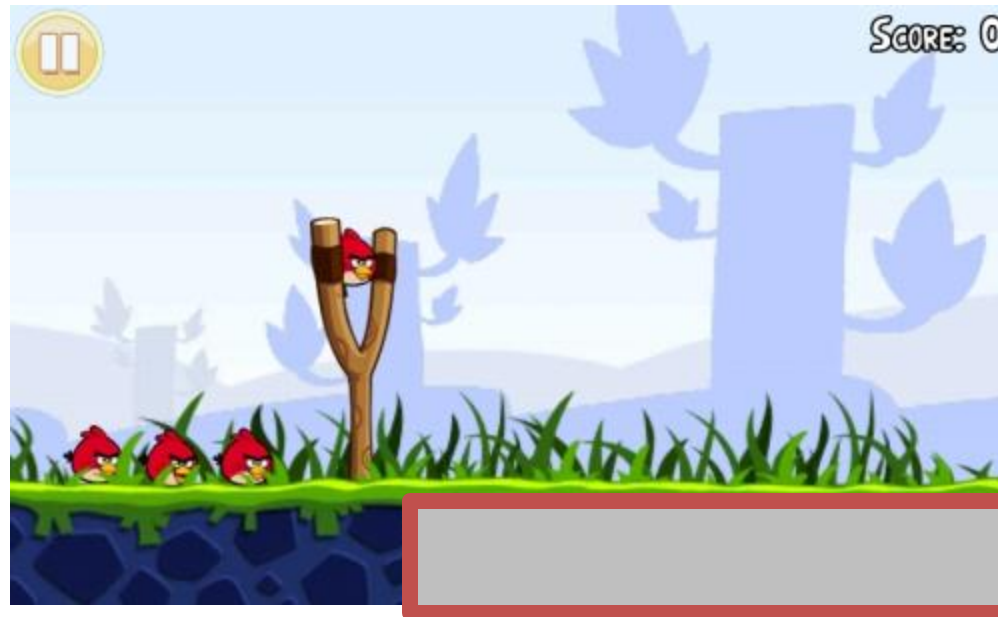




Which ad to place and how much to charge?



Which ad to place and how much to charge?



\$0.03 / click

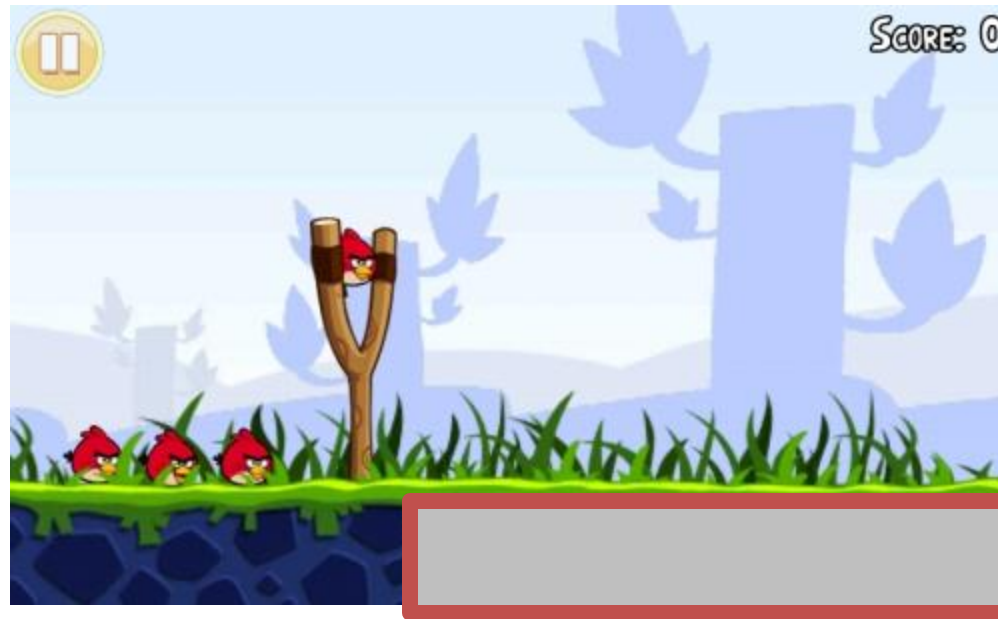


\$0.02 / click



\$0.01 / click

Which ad to place and how much to charge?



\$0.03 / click

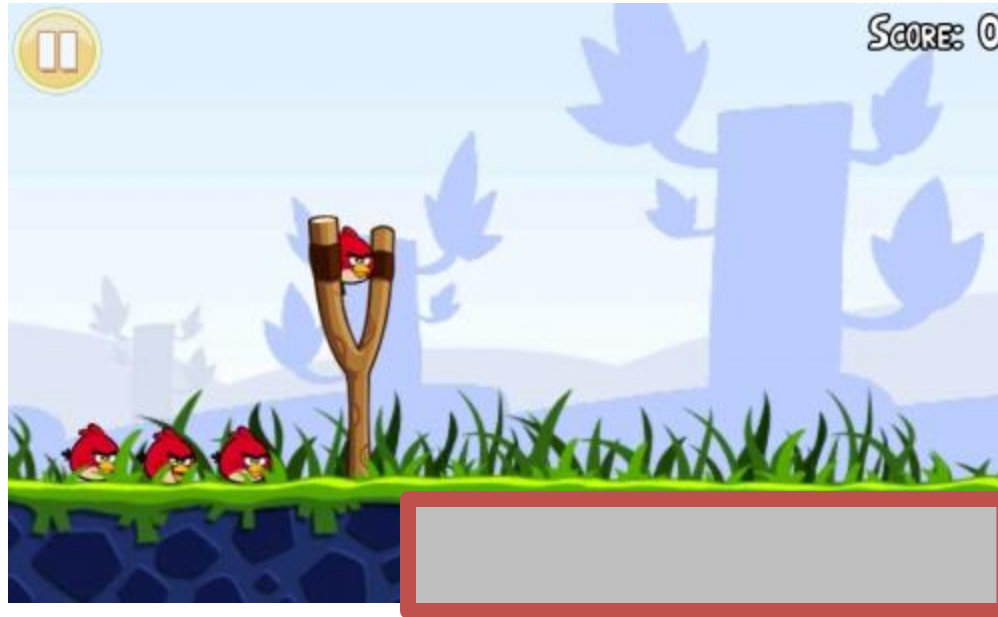


\$0.02 / click



\$0.01 / click

Which ad to place and how much to charge?



\$0.03 / click

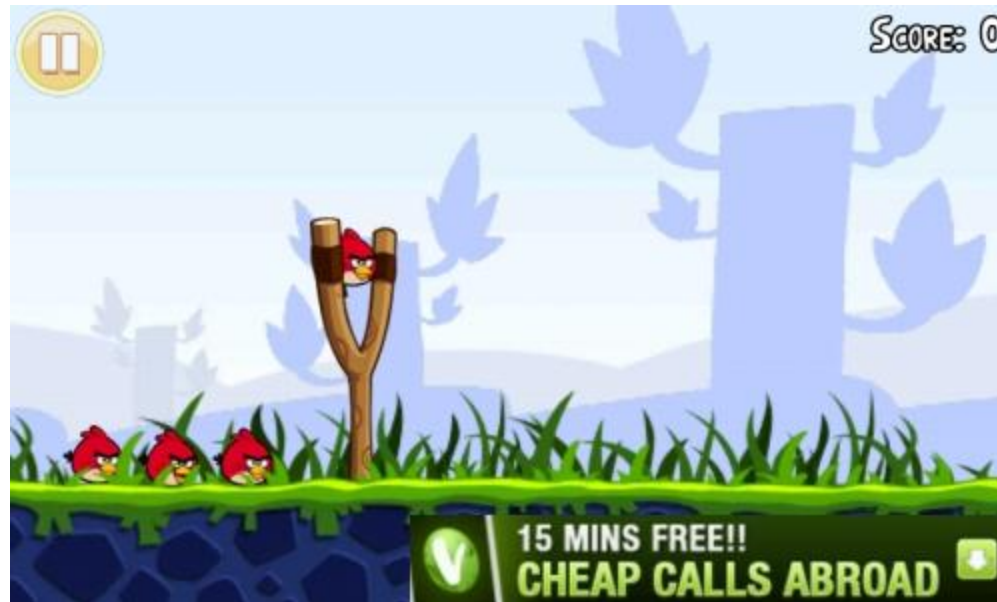


\$0.02 / click



\$0.01 / click

Which ad to place and how much to charge?



Pays \$0.02 / click (second highest bid)

Selling Display Ads



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The New York Times

Wednesday, April 11, 2012 Last Update: 11:27 AM ET



imagination at work

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U.S. Sues Apple and Major Publishers Over E-Book Pricing

By JULIE BOSMAN 5 minutes ago

The Justice Department filed an antitrust lawsuit charging that Apple and others colluded to raise the price of e-books. Several publishers have reportedly agreed to a settlement.

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CAMPAIGN 2012

Evangelicals Edge Toward Romney, but Cautiously

By ERIK ECKHOLM 10:03 AM ET

With Rick Santorum out of the race, evangelicals may



Mike Groll/Associated Press; Librado Romero/The New York Times; Mike Groll/Associated Press; Nathaniel Brooks for The New York Times

FROM THE MAGAZINE

When Is a Flip Not a Flop?

By BILL KELLER

The four Republicans who broke with their party on New York's gay-marriage law were supposedly marked for electoral death. But that's not how it is working out.

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OP-ED: JOSEPH P. KENNEDY II

The High Cost of Gambling on Oil

Cutting out the middlemen could drive down the price of oil by as much as 40 percent and that of gasoline by as much as \$1 a gallon.

- Dowd: State of Cool
- Friedman: Not Romney
- Douhat: Liberalism of Fear
- 'Buffett Rule' and Obama
- To Stop the Killing, Deal With Assad
- Why Do Americans Resist a 'Right to Die'?

MARKETS »

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+0.93%	+0.77%	+1.10%

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ENERGY FOR TOMORROW

Selling Display Ads

Single ad slot  Second-price auction

Selling Display Ads

Single ad slot  Second-price auction

- simple
- easy to bid (truthful)
- maximizes welfare
- maximizes revenue with optimal reserve r

Selling Display Ads

Single ad slot  Second-price auction

Theorem (Myerson'81)

The revenue-optimal auction* for a single item is the second price auctions with a reserve price.

* under suitable assumptions

Usually we have many ad slots ...



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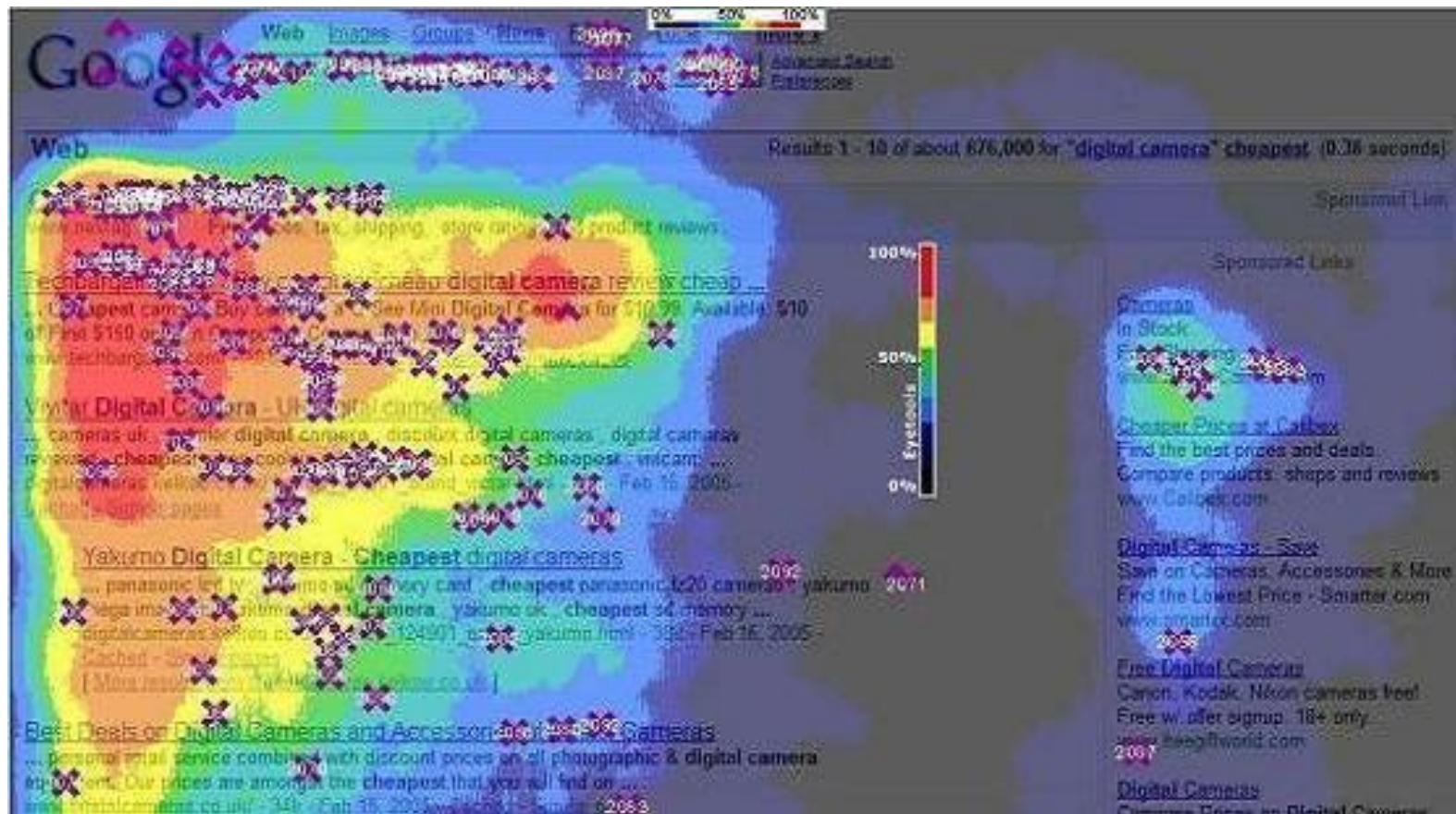
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... and not all of them are equal



... and not all of them are equal

Position Auctions



auctioning positions on webpages

How to place ads and how to charge for them with multiple positions.

Our Results

- 1) **Without uncertainty**: GSP revenue is at least $\frac{1}{2}$ of a natural VCG-like benchmark
- 2) **With uncertainty (Bayesian setting)**: GSP with appropriate reserve price extracts constant fraction of the optimal revenue
- 3) **Revenue and Welfare**:
What are the trade-offs?

Our Results

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What are the trade-offs?

Related Work

Position Auctions model due to [Edelman, Ostrovsky, Schwarz] and [Varian], who study revenue of GSP for a particular class of equilibria.

Related Work

Position Auctions model due to [Edelman, Ostrovsky, Schwarz] and [Varian], who study revenue of GSP for a particular class of equilibria.

In contrast, we study revenue of GSP over all equilibria in settings with uncertainty.

Position Auctions Model

- **n** advertisers and **n** slots
- slot *i* has click through rate α_i
- values per click $v_i \sim F$ i.i.d.
- distributions F are nice*
- reserve price **r**

* **nice** = regular, i.e., revenue curve $R(q) = q \cdot F^{-i}(1 - q)$
is concave, e.g., uniform, exponential, normal, ...

Position Auctions Model

- Strategy for i : a bidding function $b_i(v_i)$
- Assume non-overbidding $b_i(v_i) \leq v_i$
- Bidding functions form a Bayes-Nash equilibrium, i.e., no player benefits from changing bids even after learning their valuation

Position Auctions Model

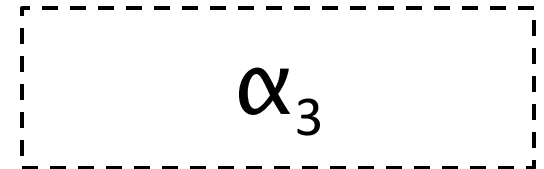
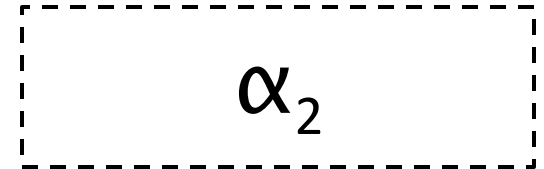
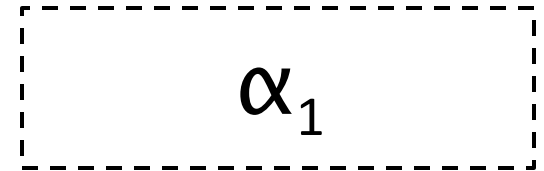
$$v_1 \sim F_1$$



$$v_2 \sim F_2$$



$$v_3 \sim F_3$$



Position Auctions Model

$$v_1 \sim F_1$$



$$b_1(v_1)$$

$$v_2 \sim F_2$$



$$b_2(v_2)$$

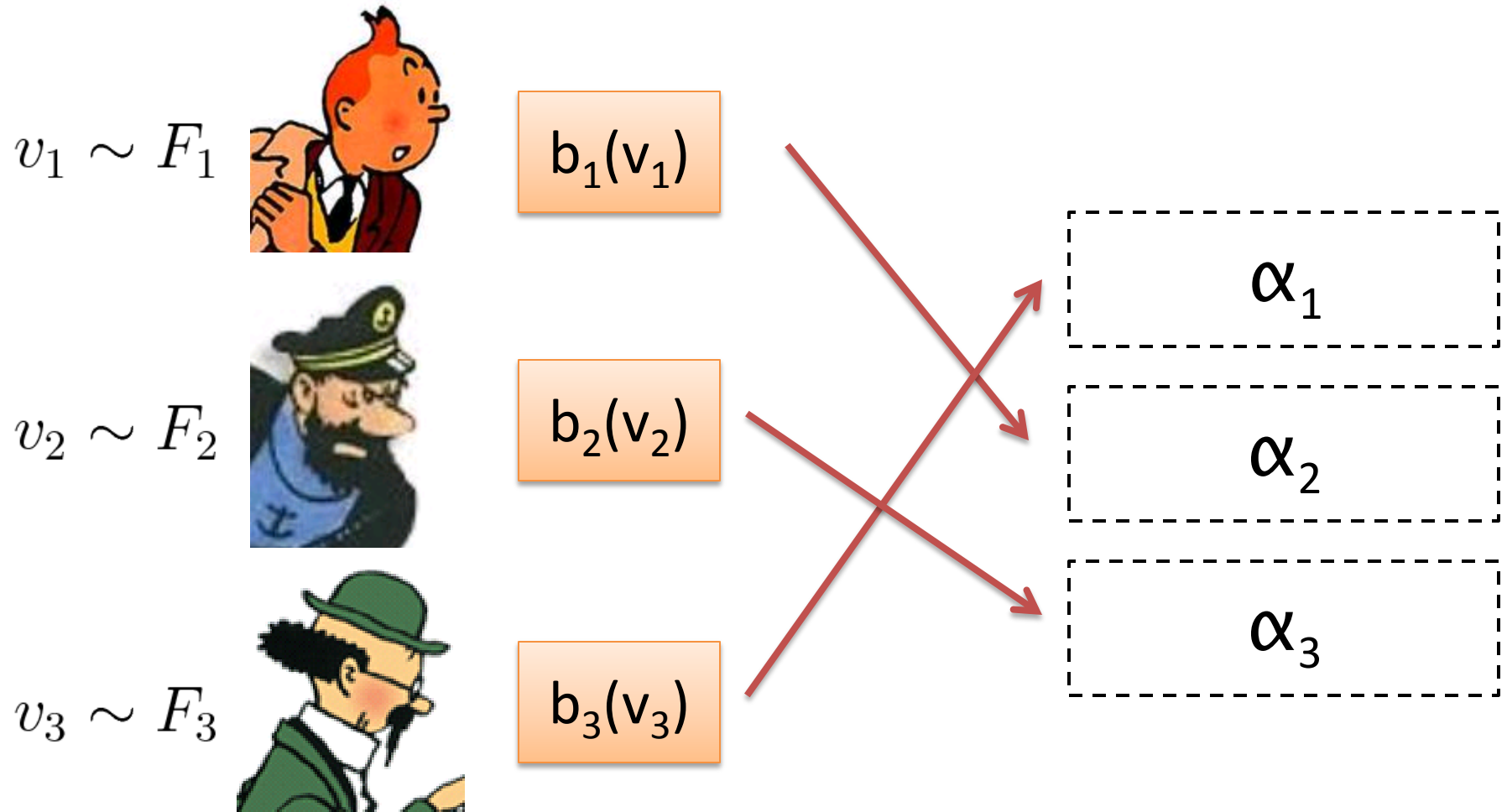
$$v_3 \sim F_3$$



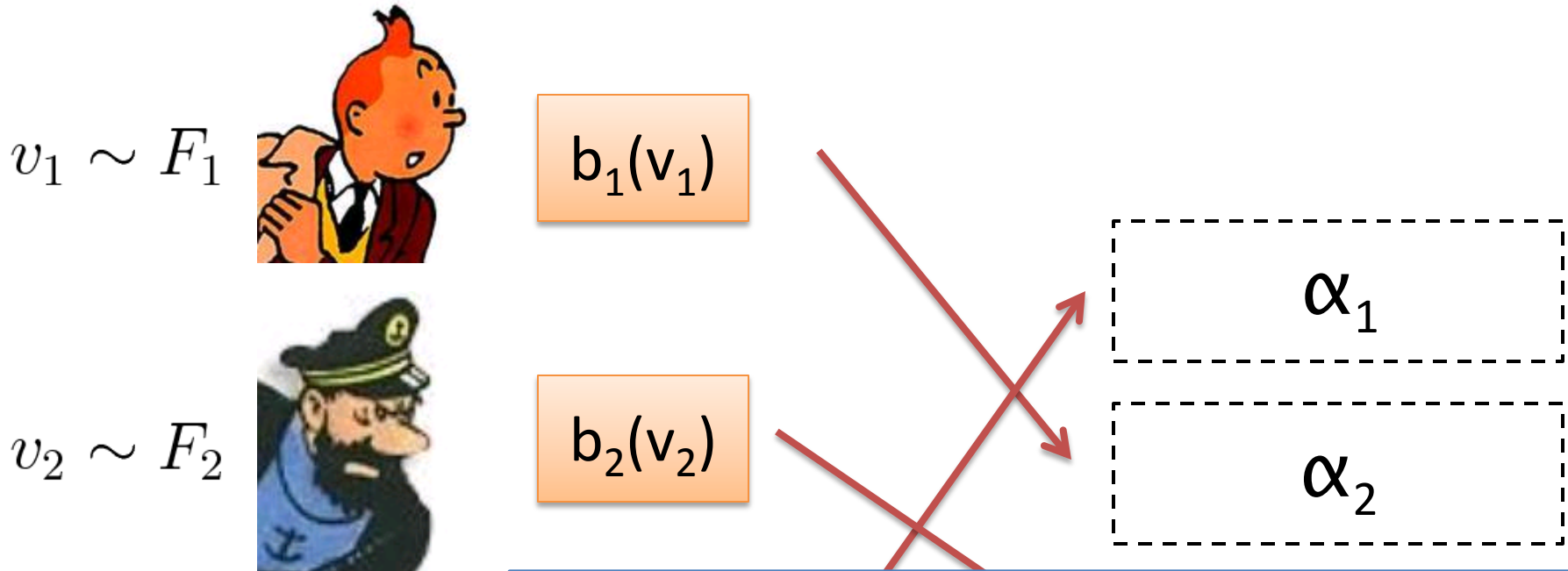
$$b_3(v_3)$$

$$\alpha_1$$
$$\alpha_2$$
$$\alpha_3$$

Position Auctions Model



Position Auctions Model



- Remove bids below r
- Sort by bid
- Charge next highest bid or r [GSP]
- Charge a complicated function [VCG]

Our Main Result

Thm: If v_i are iid according to a regular distribution, then GSP with the appropriate reserve price r extracts at least a $1/6$ fraction of the revenue of the optimal mechanism.

$b_1(\cdot), \dots, b_n(\cdot)$ is a Bayes-Nash equilibrium of GSP_r then its revenue $\mathcal{R}(b)$ is at least $1/6$ of the revenue of any mechanism and any equilibrium of this mechanism.

Our Main Result

Thm: If v_i are iid according to a regular distribution, then GSP with the appropriate reserve price r extracts at least a $1/6$ fraction of the revenue of the optimal mechanism.

Why is this good ?

$1/6$ is an absolute worst-case theoretical guarantee.

It assumes very little about distributions.

The guarantee doesn't depend on # of players, alphas, or characteristics of the distribution (besides regularity).

Our Main Result

Application:

We give guidelines for setting reserve prices in GSP.

Myerson's Thm prescribes how to set reserves on VCG.

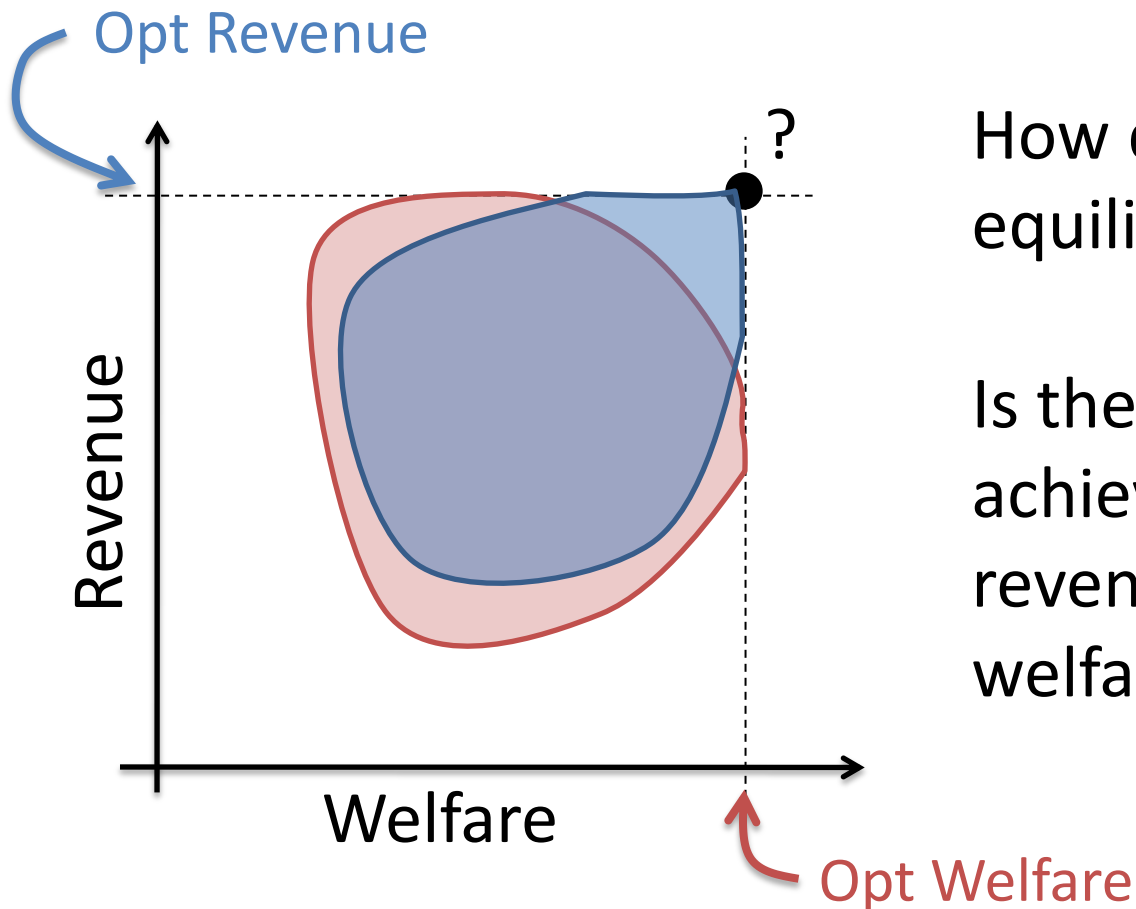
[see Ostrovsky and Schwarz, EC'11]

Ingredients of the proof:

- 1) study two cases:
 - (i) revenue is distributed among many slots
 - (ii) most of the revenue comes from one slot
- 2) passing to virtual values
- 3) prophet inequalities

Revenue-Efficiency Trade offs

Now in the non-Bayesian world, where v_i are fixed:



How does the plot of all equilibria look like?

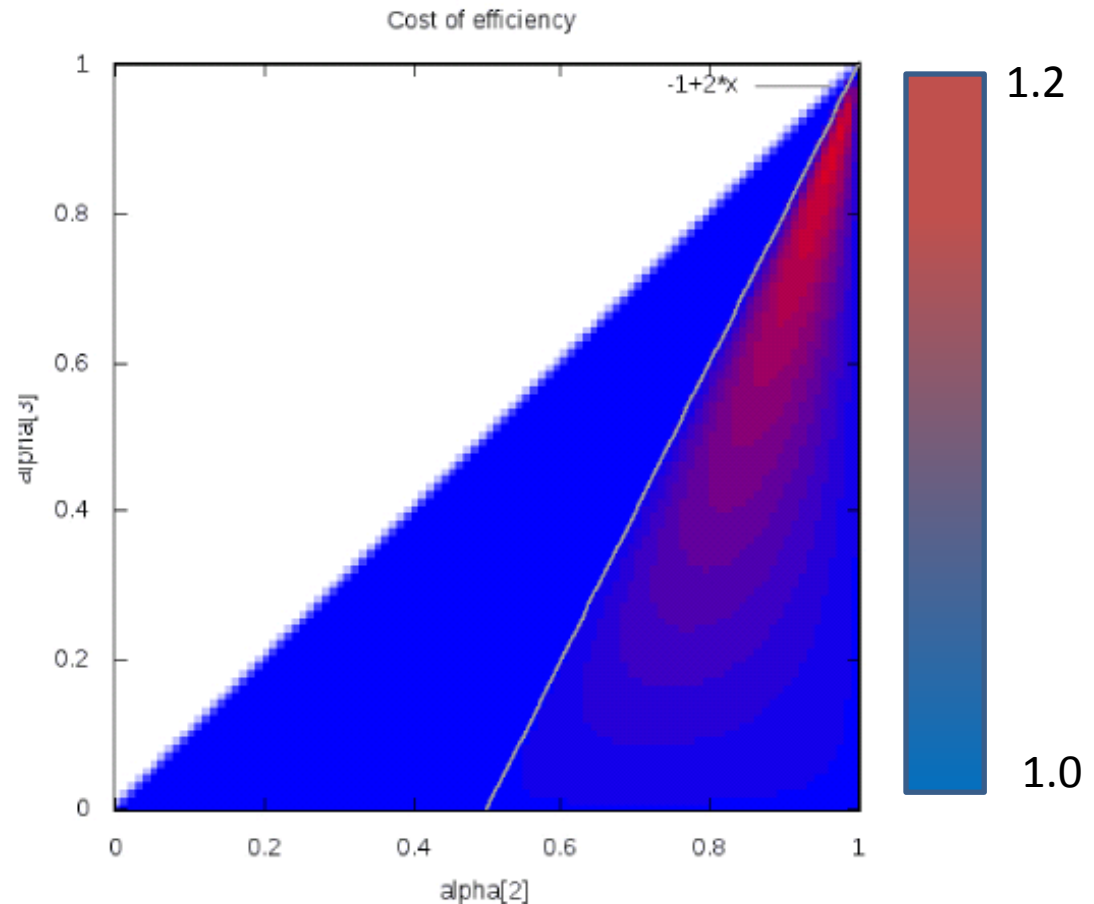
Is there an equilibrium achieving maximum revenue and social welfare ?

Revenue-Efficiency Trade offs

Observation: The **revenue optimal** equilibrium might be **inefficient**.

Cost of efficiency: ratio between revenue-optimal equilibrium and revenue-optimal efficient equilibrium. For vector:

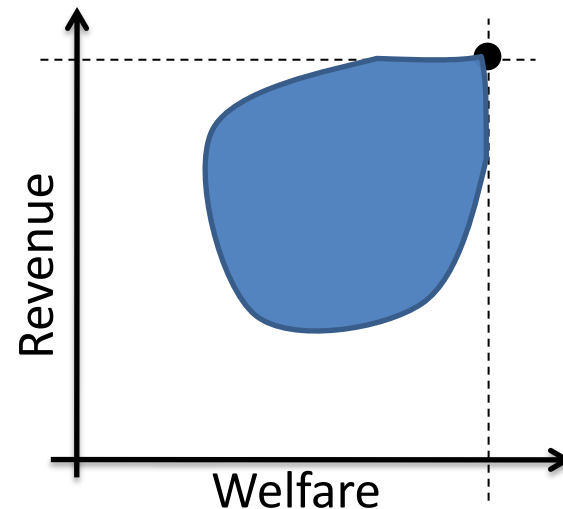
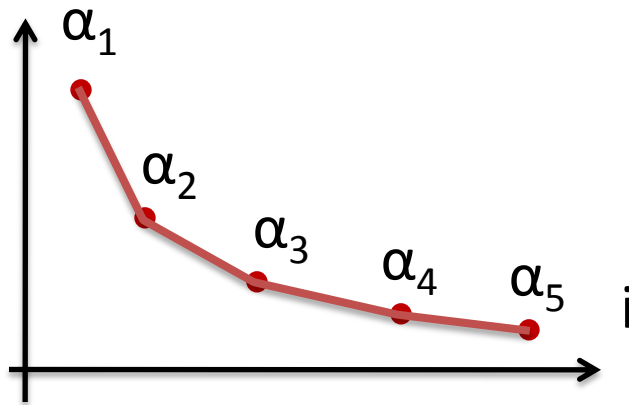
$$\alpha = [1, \alpha_2, \alpha_3]$$



Revenue-Efficiency Trade offs

Observation: The **revenue optimal** equilibrium might be **inefficient**.

Thm 3: If CTR α are convex, then the revenue-optimal equilibrium is efficient.



Revenue-Efficiency Trade offs

Thm 3: If CTR α are convex, then the revenue-optimal equilibrium is efficient.

Proof Idea:

- Structure of revenue in inefficient equilibria
- Local improvement proof

Conclusion

- Bounds on revenue extracted by GSP
 - Full Information
 - Bayesian setting
 - Revenue x Welfare tradeoffs

Conclusion

- Bounds on revenue extracted by GSP
 - Full Information
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 - Revenue x Welfare tradeoffs

Thanks !

